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Five Dimensional Domain Walls in a Scalar-Tensor Theory of Gravitation

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Abstract Five dimensional Kaluza-Klein space-time is considered in the presence of thick domain walls in the framework of scalar-tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A 113:467, 1986). Exact cosmological model, which represents a stiff domain wall, is presented. Some physical and kinematical properties of the model are also discussed.

Keywords Scalar-tensor theory · Domain walls · Kaluza-Klein space-time

1 Introduction

In recent years there has been a lot of interest to know the exact physical situation at very early stages of evolution of the universe. The phase transitions in the early universe, due to spontaneous breaking of discrete symmetry, could have produced the topological defects such as domain walls, cosmic strings and monopoles [1]. These topological objects have an important role in the formation of the universe, in particular strings and domain walls have exited a lot of interest among researchers due to their peculiar and interesting gravitational effects. Several authors have investigated the gravitational effects of cosmic strings in general relativity and in alternative theories of gravitation. Recently, the study of thick domain walls and space times associated with them have received considerable attention due to their vital role in structure formation in the universe [2]. Goetz [3], Mukharjee [4], Widrow [5], Vilenkin [6, 7], Isper and Sikivie [8], Wang [9], Rahaman et al. [10], Rahaman and Kalam [11], Chakraborty et al. [12] and Rahaman and Chakraborty [13] are some of the authors who have investigated several aspects of domain walls in four and five dimensions in general relativity, while Rahaman et al. [14], Rahaman [15], Rahaman and Mukhar-

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jee [16] and Reddy and Rao [17] have discussed thick domain walls in Lyra [18] geometry. Very recently there has been much interest on gravitational effects of cosmic strings and domain walls in alternative theories of gravitation especially in Brans-Dicke [19] and Saez-Ballester [20] scalar-tensor theories of gravity in view of the latest inflationary models [21–23].

Brans-Dicke theory introduces a scalar field ϕ, in addition to the familiar general relativistic metric tensor field g_{ij}, which has the dimension of the inverse of a gravitational constant which interacts equally with all forms of matter (with the exception of electromagnetism). Saez and Ballester [20] have formulated a scalar-tensor theory of gravitation in which the metric is coupled with dimension less scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. Inspite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non-flat FRW cosmologies.

Reddy [24], Reddy et al. [25] and Reddy and Naidu [26] have discussed some string cosmological models in Saez-Ballester scalar-tensor theory in four dimensions while Reddy and Naidu [27] have obtained five dimensional string cosmological models in this theory. Very recently Adhav et al. [28] have investigated axially symmetric non-static domain walls in Brans-Dicke and Saez-Ballester scalar-tensor theories of gravitation in four dimensional space-time.

In this paper, we discuss the domain walls in Kaluza-Klein five dimensional space-time in the frame work of Saez-Ballester scalar-tensor theory of gravitation. The study of higher dimensional space-time is important because of the underlying idea that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. This fact has attracted many researchers to the field of higher dimensions [29, 30]. Solutions of gravitational field equations in higher dimensional space time are believed to be of physical relevance, possibly, at the early times before the universe has undergone compactification transition.

2 Metric and Field Equations

We consider five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - R^2(dx^2 + dy^2 + dz^2) - A^2 dm^2 \tag{1}$$

Unlike Wesson [31], the fifth coordinate is taken to be space-like and the metric coefficients R and A are assumed to be functions of time only.

The field equations given by Saez and Ballester for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n(\phi_{,i}\phi_{,j} - 1/2 g_{ij}\phi_{,k}\phi^{,k}) = T_{ij} \tag{2}$$

and the scalar field satisfies the equation

$$2\phi^n \phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

where $G_{ij} = R_{ij} - 1/2 g_{ij} R$ is the Einstein tensor, ω and n are constants, T_{ij} is the energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively. Also

$$T^{ij}_{;j} = 0 \tag{4}$$

is a consequence of the field equations (2) and (3).

There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy momentum tensor describing a self-interacting scalar field contained in a potential $V(\psi)$ given by

$$\psi_{,i}\psi_{,j} - g_{ij}[1/2\psi_{,k}\psi^{,k} - V(\psi)] \tag{5}$$

The second approach is to assume the stress energy tensor for a domain wall in the form

$$\begin{aligned} T_{ij} &= \rho(g_{ij} + \omega_i\omega_j) + p_1\omega_i\omega_j, \\ \omega^i\omega_i &= -1 \end{aligned} \tag{6}$$

where ρ is the energy density of the wall, p_1 is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction.

Here we use the second approach to study the thick domain walls in Saez-Ballester theory. In the commoving coordinate system we have from (6) stress energy components for domain walls

$$T_0^0 = T_1^1 = T_2^2 = T_3^3 = \rho, \qquad T_4^4 = -p_1 \tag{7}$$

The field equations (2)–(4) of Saez-Ballester theory for the metric (1) with the help of (6) and (7) can be written as

$$3\frac{R_4^2}{R^2} + 3\frac{R_4A_4}{RA} - \frac{\omega}{2}\phi^n\phi_4^2 = \rho \tag{8}$$

$$2\frac{R_{44}}{R} + \frac{R_4^2}{R^2} + 2\frac{R_4A_4}{RA} + \frac{A_{44}}{A} + \frac{\omega}{2}\phi^n\phi_4^2 = \rho \tag{9}$$

$$3\frac{R_{44}}{R} + 3\frac{R_4^2}{R^2} + \frac{\omega}{2}\phi^n\phi_4^2 = -p_1 \tag{10}$$

$$\phi_{44} + \phi_4\left(3\frac{R_4}{R} + \frac{A_4}{A}\right) + \frac{n}{2}\frac{\phi_4^2}{\phi} = 0 \tag{11}$$

$$\rho_4 - (\rho + p_1)\frac{A_4}{A} = 0 \tag{12}$$

where suffixe 4 denotes differentiation with respect to t.

3 Cosmic Domain Walls

Here we have four independent field equations (8)–(11) connecting five unknowns R, A, ϕ, ρ and p_1. Hence to get a determinate solution one has to assume physical or mathematical conditions. Here we assume

$$\rho = p_1 \tag{13}$$

This condition is analogous to the stiff fluid (self-gravitating fluid) equation of state in general relativity. Since the field equations are highly non-linear we also assume an analytic relation between the metric coefficients

$$A = \mu R^n \tag{14}$$

Now, using (13) and (14) in the field equations (8)–(11) we obtain

$$\begin{aligned} R &= [(3n+5)(c_1 t + c_2)]^{1/(3n+5)} \\ A &= \mu[(3n+5)(c_1 t + c_2)]^{n/(3n+5)} \end{aligned} \tag{15}$$

where c_1 and c_2 are constants of integration. After a suitable choice of coordinates and constants of integration, the five dimensional model can be written as

$$ds^2 = dt^2 - [(3n+5)T]^{2/(3n+5)}(dx^2 + dy^2 + dz^2) - \mu^2[(3n+5)T]^{2n/(3n+5)} dm^2 \tag{16}$$

4 Some Physical Properties of the Model

The model given by (16) can be viewed as a five dimensional self-gravitating or stiff domain wall in Saez-Ballester scalar-tensor theory of gravitation. The model has no initial singularity. The pressure p_1 and density ρ in the universe (16) are given by

$$\rho = p_1 = \frac{\omega}{2}[(3n+5)T]^{-2(n+3)} - (3n^2 + 8n + 7)[(3n+5)T]^{-2} \tag{17}$$

and the scalar field ϕ in the model is given by

$$\phi^{\frac{n}{2}+1} = \phi_0 + \frac{1}{2}[(3n+5)T]^{-(n+2)} \tag{18}$$

The kinematical parameters of the model (16) have the following expressions

Spatial volume: $V^3 = \mu[(3n+5)T]^{(n+3)/(3n+5)}$

Expansion scalar: $\theta = \frac{1}{3}u^i_{;i} = \left[\frac{n+3}{3n+5}\right]\frac{1}{3T}$

Shear scalar: $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{54}\left[\frac{n+3}{3n+5}\right]^2 \frac{1}{T^2}$

The deceleration parameter [32]: $q = -\frac{3}{\theta^2}\left[\theta_{;i}u^i + \frac{1}{3}\theta^2\right] = 8$

The energy density ρ and the pressure p_1 of the domain wall tend to zero and the scalar field ϕ becomes constant as T in creases indefinitely and they also possess singularity at $T = 0$. The model (16) is quite similar to the five dimensional Reddy string [29, 30] in Saez-Ballester theory. For this model the spatial volume tends to infinity while the expansion scalar and shear scalar tend to zero as $T \to \infty$. The positive value of the deceleration parameter shows that the model decelerates in the standard way.

5 Conclusions

In this paper we have obtained a five dimensional cosmic domain wall in Saez-Ballester [20] scalar-tensor theory of gravitation. To get a determinate solution we have assumed stiff fluid condition and a relation between the metric coefficients. The model obtained can be viewed as a five dimensional self-gravitating or stiff domain wall in this theory. This model will help for a better understanding of the structure formation at the early stages of evolution of the universe.

References

1. Kibble, T.W.B.: J. Phys. A. Math. Gen. **9**, 1387 (1976)
2. Hill, C.T., Schnam, D.N., Fry, J.N.: Comments Nucl. Part. Phys. **19**, 25 (1989)
3. Goetz, G.: J. Math. Phys. **31**, 2683 (1990)
4. Mukherjee, M.: Class. Quantum Gravity **10**, 131 (1993)
5. Widrow, L.M.: Phys. Rev. D **40**, 1002 (1989)
6. Vilenkin, A.: Phys. Rev. D **23**, 852 (1981)
7. Vilenkin, A.: Phys. Lett. B **133**, 177 (1983)
8. Isper, J., Sikivie, P.: Phys. Rev. D **30**, 712 (1984)
9. Wang, A.: Mod. Phys. Lett. A **90**, 3605 (1994)
10. Rahaman, F., Chakraborty, S., Kalam, M.: Astrophys. Space Sci. **282**, 391 (2002)
11. Rahaman, F., Kalam, M.: Pramana J. Phys. **58**, 127 (2002)
12. Chakraborty, S., Rahaman, F., Biswas, L.: Astrophys. Space Sci. **274**, 851 (2000)
13. Rahaman, F., Chakraborty, S.: Gen. Relativ. Gravit. **32**, 1757 (2000)
14. Rahaman, F., Chakraborty, S., Hossain, M., Begum, N., Bera, J.: Indian J. Phys. B **76**, 747 (2002)
15. Rahaman, F.: Astrophys. Space Sci. **282**, 625 (2002)
16. Rahaman, F., Mukherjee, R.: Astrophys. Space Sci. **288**, 493 (2003)
17. Reddy, D.R.K., Rao, M.V.S.: Astrophys. Space Sci. **302**, 157 (2006)
18. Lyra, G.: Math. Z. **54**, 52 (1951)
19. Brans, C.H., Dicke, R.H.: Phys. Rev. **124**, 925 (1961)
20. Saez, D., Ballester, V.J.: Phys. Lett. A **113**, 467 (1986)
21. Mathiazhagan, C., Johri, V.B.: Class. Quantum Gravity **10**, 131 (1993)
22. Linde, A.: Phys. Lett. B **238**, 160 (1990)
23. Pimental, L.O.: Mod. Phys. Lett. A **12**, 1865 (1997)
24. Reddy, D.R.K.: Astrophys. Space Sci. **286**, 359 (2003)
25. Reddy, D.R.K., Naidu, R.L., Rao, V.U.M.: Astrophys. Space Sci. **302**, 157 (2006)
26. Reddy, D.R.K., Naidu, R.L.: Astrophys. Space Sci. **312**, 99 (2007)
27. Reddy, D.R.K., Naidu, R.L.: Astrophys. Space Sci. **307**, 395 (2007)
28. Adhav, K.S., Nimkar, A.S., Naidu, R.L.: Astrophys. Space Sci. **312**, 165 (2007)
29. Witten, E.: Phys. Lett. B **144**, 351 (1984)
30. Appelquist, T., Chodos, A., Freund, P.G.O.: Modern Kaluza-Klein Theories. Addison-Wesley, Reading (1987)
31. Wesson, P.S.: Astron. Astrophys. **119**, 1 (1983)
32. Feinstein, A., Ibanez, J., Labarge, D.: J. Math. Phys. **36**, 4962 (1995)